Comment on "Potential Flow around Axisymmetric Bodies: Direct and Inverse Problems"

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THE authors attribute their difficulty with the von Kármán method to the loss of accuracy in the computation of

$$\psi_{ij}=\sqrt{(x_i-x_j-\ell_j)^2+r_i^2}-\sqrt{(x_i-x_j)^2+r_i^2} \tag{1}$$

for small ℓ_j.

I would suggest that this difficulty could be circumvented by using the mathematically equivalent form

$$\psi_{ij}=\frac{\ell_j^2-2\ell_j(x_i-x_j)}{\sqrt{(x_i-x_j-\ell_j)^2+r_i^2}+\sqrt{(x_i-x_j)^2+r_i^2}} \tag{2}$$

obtained by multiplying through by $(\sqrt{R_1}+\sqrt{R_2})/\sqrt{R_1}+\sqrt{R_2})$ and performing the subtraction in the numerator analytically. This gives a reasonably well behaved numerator and a denominator that is the sum of two positive terms.

It would be interesting to see what this simple modification of the computer program does to the results.

References

[1]Zedan, M. F. and Dalton, C., "Potential Flow around Axisymmetric Bodies: Direct and Inverse Problems," *AIAA Journal*, Vol. 16, March 1978, pp. 242-250.

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Reply by Authors to W. Squire

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THE authors appreciate the interest shown in their work by Prof. Squire. We had, in fact, already tried his suggestion of recasting the expression for ψ_{ij} (an unnumbered equation on p. 245 of Ref. 1) into an equivalent form. Figure 1 shows the result of making the suggested change in the expression for ψ_{ij} for an airfoil-type axisymmetric body of fineness ratio $\simeq 2.91$. The figure shows that the exact solution and the solution obtained by linearly varying axial singularities are essentially the same. The results computed by the expression in the paper for the von Kármán method are improved slightly by the change suggested by Squire. However, agreement between the "improved" results and the exact solution is still lacking in the tail region.

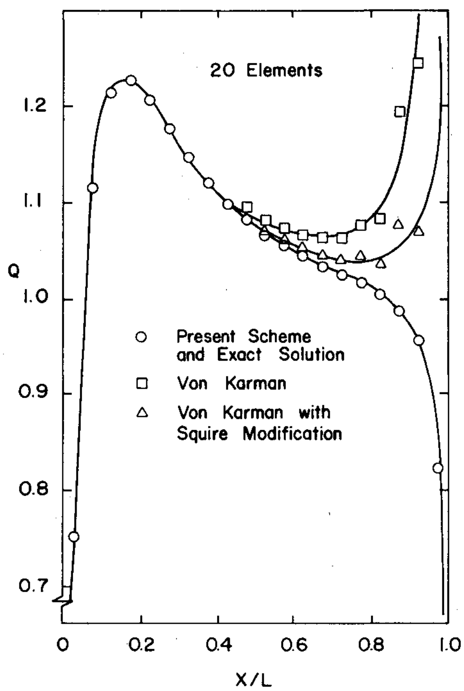

Fig. 1 Velocity distribution for airfoil-type axisymmetric body of $F \cdot R \cong 2.91$.

Oberkampf and Watson[2] and Zedan and Dalton[1] observed that the matrix of the von Kármán method is ill-conditioned. Slight errors in the elements of the matrix or in the vector (right-hand side of the equations) can lead to substantial changes in the solution. With the suggested modification, the numerator of ψ_{ij} is reasonably well behaved so long as $(x_i-x_j) \gg \ell_j/2$. However, (x_i-x_j) becomes of the order of $\ell_j/2$ for $j=i$ or $i+1$. The problem of the loss of accuracy appears again, especially if the element length is small (large number of elements). In effect, the suggested modification reduces the number of inaccurate elements in the matrix (this explains the slight improvement). However, with an ill-conditioned matrix, this does not solve the problem; a slight error even in one element can cause considerable change in the solution. Forsythe et al.,[3] using a system of two equations consisting of a matrix with exact elements, showed that the solution can be very different when using computers of different accuracy if the matrix is ill-conditioned.

Therefore, it appears that ill-conditioned matrices should be avoided by changing the structure of the solution (equivalent to our effort[1]) instead of trying to improve the computational accuracy, as suggested by Squire.

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[1]Zedan, M. F. and Dalton, C., "Potential Flow around Axisymmetric Bodies: Direct and Inverse Problems," *AIAA Journal*, Vol. 16, March 1978, pp. 242-250.

[2]Oberkampf, W. L., and Watson, L. E., "Incompressible Potential Flow Solutions for Arbitrary Bodies of Revolution," *AIAA Journal*, Vol. 12, March 1974, pp. 409-411.

[3]Forsythe, G. E., Malcolm, M. A., and Moler, C. B., *Computer Methods for Mathematical Computations*, Prentice-Hall, Englewood Cliffs, N.J., 1977, pp. 38-39.